

NON-COMMUTATIVE RESIDUE OF PROJECTIONS IN BOUTET DE MONVEL'S CALCULUS

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ABSTRACT. Using results by Melo, Nest, Schick, and Schrohe on the K -theory of Boutet de Monvel's calculus of boundary value problems, we show that the non-commutative residue introduced by Fedosov, Golse, Leichtnam, and Schrohe vanishes on projections in the calculus.

This partially answers a question raised in a recent collaboration with Grubb, namely whether the residue is zero on sectorial projections for boundary value problems: This is confirmed to be true when the sectorial projections is in the calculus.

1. INTRODUCTION

Boutet de Monvel [2] constructed a calculus, often called the Boutet de Monvel calculus (or algebra), of pseudodifferential boundary operators on a manifold with boundary. It includes the classical differential boundary value problems as well of the parametrices of the elliptic elements:

Let X be a compact n -dimensional manifold with boundary ∂X ; we consider X as an embedded submanifold of a closed n -dimensional manifold \tilde{X} . Denote by X° the interior of X . Let E and F be smooth complex vector bundles over X and ∂X , respectively, with E the restriction to X of a bundle \tilde{E} over \tilde{X} .

An operator in Boutet de Monvel's calculus — a (polyhomogeneous) Green operator — is a map A acting on sections of E and F , given by a matrix

$$(1.1) \quad A = \begin{pmatrix} P_+ + G & K \\ T & S \end{pmatrix} : \begin{array}{c} C^\infty(X, E) \\ \oplus \\ C^\infty(\partial X, F) \end{array} \rightarrow \begin{array}{c} C^\infty(X, E) \\ \oplus \\ C^\infty(\partial X, F) \end{array},$$

where P is a pseudodifferential operator (ψ do) on \tilde{X} with the transmission property and P_+ is its truncation to X :

$$(1.2) \quad P_+ = r^+ P e^+, \quad r^+ \text{ restricts from } \tilde{X} \text{ to } X^\circ, \quad e^+ \text{ extends by } 0.$$

G is a singular Green operator, T a trace operator, K a Poisson operator, and S a ψ do on the closed manifold ∂X . See [2], Grubb [6], or Schrohe [13] for details.

Fedosov, Golse, Leichtnam, and Schrohe [4] extended the notion of non-commutative residue known from closed manifolds (cf. Wodzicki [14], [15], and Guillemin [9]) to the algebra of Green operators. The noncommutative residue of A from (1.1) was defined to be

$$(1.3) \quad \text{res}_X(A) = \int_X \int_{S_x^* X} \text{tr}_E p_{-n}(x, \xi) dS(\xi) dx \\ + \int_{\partial X} \int_{S_{x'}^* \partial X} [\text{tr}_E(\text{tr}_n g)_{1-n}(x', \xi') + \text{tr}_F s_{1-n}(x', \xi')] dS(\xi') dx'.$$

Here tr_E and tr_F are traces in $\text{Hom}(E)$ and $\text{Hom}(F)$, respectively; $dS(\xi)$ (resp. $dS(\xi')$) denotes the surface measure divided by $(2\pi)^n$ (resp. $(2\pi)^{n-1}$); $\text{tr}_n g$ is the normal trace of g ; and the subscripts $-n$ and $1-n$ indicate that we only consider the homogeneous terms of degree $-n$ resp. $1-n$. Also, a sign error in [4] has been corrected, cf. Grubb and Schrohe [8, (1.5)].

It is well-known [14] that on a closed manifold, the noncommutative residue of a classical ψ do projection (or idempotent) is zero. In the present paper we wish to show that the same holds in the case of Green operators. We will use K -theoretic arguments (in a C^* -algebra setting) to effectively reduce the problem to the known case of closed manifolds.

In our recent collaboration with Grubb [5] we studied certain spectral projections: For the realization $B = (P + G)_T$ of an elliptic boundary value problem $\{P_+ + G, T\}$ of order $m > 0$ with two spectral cuts at angles θ and φ , one can define the *sectorial projection* $\Pi_{\theta, \varphi}(B)$. It is a (not necessarily self-adjoint) projection whose range contains the generalized eigenspace of B for the sector $\Lambda_{\theta, \varphi} = \{re^{i\omega} \mid r > 0, \theta < \omega < \varphi\}$ and whose nullspace contains the generalized eigenspace for $\Lambda_{\varphi, \theta+2\pi}$. It was considered earlier by Burak [3], and in the boundary-less case by Wodzicki [14] and Ponge [12].

In general this operator is not in Boutet de Monvel's calculus, but we showed that it has a residue in a slightly more general sense. The question was posed whether this residue vanishes.

The question of the non-commutative residue of projections is particularly interesting in the context of zeta-invariants as discussed by Grubb [7] and in [5]: The *basic zeta value* $C_{0, \theta}(B)$ for the realization B of a boundary value problem is defined via a choice of spectral cut in the complex plane; the

difference in the basic zeta value based on two spectral cut angles θ and φ is then given as the non-commutative residue of the corresponding sectorial projection:

$$(1.4) \quad C_{0,\theta}(B) - C_{0,\varphi}(B) = \frac{2\pi i}{m} \operatorname{res}_X(\Pi_{\theta,\varphi}(B)).$$

Our results here show that the dependence of $C_{0,\theta}(B)$ upon θ is trivial whenever the projection $\Pi_{\theta,\varphi}(B)$ lies in Boutet de Monvel's calculus.

It should be noted that the litterature in functional analysis and PDE-theory often uses “projection” as a synonym for idempotent, while C^* -algebraists furthermore require that projections are self-adjoint; we will try to avoid confusion by explicitly using the term “ ψ do projection” for the idempotent operators here.

2. PRELIMINARIES AND NOTATION

We employ Blackadar's [1] approach to K -theory: A pre- C^* -algebra B is called local if it, as a subalgebra of its C^* -completion \overline{B} , is closed under holomorphic function calculus (and all of its matrix algebras must have this property as well). Let $\mathcal{M}_\infty(B)$ denote the direct limit of the matrix algebras $\mathcal{M}_m(B)$, $m \in \mathbb{N}$. Define $\mathcal{IP}_\infty(B) = \operatorname{Idem}(\mathcal{M}_\infty(B))$ — resp. $\mathcal{IP}_m(B) = \operatorname{Idem}(\mathcal{M}_m(B))$ — to be the set of all — resp. all $m \times m$ — idempotent matrices with entries from B . Define the relation \sim on $\mathcal{IP}_\infty(B)$ by

$$(2.1) \quad x \sim y \text{ if there exist } a, b \in \mathcal{M}_\infty(B) \text{ such that } x = ab \text{ and } y = ba.$$

If B has a unit we define $K_0(B)$ to be the Grothendieck group of the semi-group $V(B) = \mathcal{IP}_\infty(B) / \sim$. If B has no unit, we consider the scalar map from the unitization — indicated with a tilde as in \tilde{B} or B^\sim — of B to the complex numbers $s : \tilde{B} \rightarrow \mathbb{C}$ defined by $s(b + \lambda 1_{\tilde{B}}) = \lambda$, and then define $K_0(B)$ as the kernel of the induced map $s_* : K_0(\tilde{B}) \rightarrow K_0(\mathbb{C})$.

A fact that we shall use several times is that if B is local, then [1, p. 28]

$$(2.2) \quad V(B) \cong V(\overline{B}), \text{ and hence } K_0(B) \cong K_0(\overline{B}).$$

Combined with *the standard picture of K_0* this implies that

$$(2.3) \quad K_0(\overline{B}) = \{ [x]_0 - [y]_0 \mid x, y \in \mathcal{IP}_m(B), m \in \mathbb{N} \}$$

in the case where B is unital, and

$$(2.4) \quad K_0(\overline{B}) = \{ [x]_0 - [y]_0 \mid x, y \in \mathcal{IP}_m(\tilde{B}) \text{ with } x \equiv y \bmod \mathcal{M}_m(B), m \in \mathbb{N} \}$$

in the non-unital case [1].

Let \mathcal{A} denote the set of Green operators as in (1.1) of order and class zero; it defines a $*$ -subalgebra of the bounded operators on the Hilbert space $\mathcal{H} = L_2(X, E) \oplus H^{-\frac{1}{2}}(\partial X, F)$; we will denote by \mathfrak{A} its C^* -closure in $\mathcal{B}(\mathcal{H})$. \mathcal{A} is local with $\overline{\mathcal{A}} = \mathfrak{A}$, cf. Melo, Nest, and Schrohe [10], so $K_0(\mathcal{A}) \cong K_0(\mathfrak{A})$. Note that the K -theory of \mathcal{A} is independent of the specific bundles [10, Section 1.5], so for simplicity we study explicitly in this paper only the simplest trivial case $E = X \times \mathbb{C}$ and $F = \partial X \times \mathbb{C}$.

\mathcal{K} denotes the subalgebra of smoothing operators, \mathfrak{K} its C^* -closure (the ideal of compact operators). We let \mathcal{J} denote the set of elements in \mathcal{A} of the form

$$(2.5) \quad \begin{pmatrix} \varphi P\psi + G & K \\ T & S \end{pmatrix}$$

with $\varphi, \psi \in C_c^\infty(X^\circ)$, P a ψ do on \tilde{X} of order zero, and G, K, T , and S of negative order and class zero. \mathfrak{J} will be the C^* -closure of \mathcal{J} in \mathfrak{A} .

The noncommutative residue defined in [4] is a trace — a linear map that vanishes on commutators — $\text{res} : \mathcal{A} \rightarrow \mathbb{C}$, and therefore induces a group homomorphism $\text{res}_* : K_0(\mathcal{A}) \rightarrow \mathbb{C}$ such that

$$(2.6) \quad \text{res}_*([A]_0) = \text{res}_X(A)$$

for any idempotent $A \in \mathcal{A}$. Our goal is to prove the vanishing of res_* , which obviously implies that $\text{res}_X(A) = 0$ for any idempotent A .

The quotient map $q : \mathfrak{A} \rightarrow \mathfrak{A}/\mathfrak{K}$ induces an isomorphism $q_* : K_0(\mathfrak{A}) \rightarrow K_0(\mathfrak{A}/\mathfrak{K})$ [10]. The isomorphisms $K_0(\mathcal{A}) \cong K_0(\mathfrak{A}) \cong K_0(\mathfrak{A}/\mathfrak{K})$ allow us to extend the noncommutative residue: For each $[\mathcal{A} + \mathfrak{K}]_0$ in $K_0(\mathfrak{A}/\mathfrak{K})$ there is an $A \in \mathcal{IP}_\infty(\mathcal{A})$ such that $q_*[A]_0 = [\mathcal{A} + \mathfrak{K}]_0$, and we then define

$$(2.7) \quad \widetilde{\text{res}}_*[\mathcal{A} + \mathfrak{K}]_0 = \text{res}_*[A]_0 = \text{res}_X(A).$$

The map $\widetilde{\text{res}}_*$ is really just $\text{res}_* \circ q_*^{-1}$, and is thus a group homomorphism $K_0(\mathfrak{A}/\mathfrak{K}) \rightarrow \mathbb{C}$.

3. K-THEORY AND THE RESIDUE

We employ results from Melo, Schick, and Schrohe [11], in particular the fact that “each element in $K_0(\mathfrak{A}/\mathfrak{K})$ can be written as the sum of two elements, one in the range of m_* and one in the range of s'' , thus in the range of i_* ” (bottom of page 11). In other words

$$(3.1) \quad K_0(\mathfrak{A}/\mathfrak{K}) = q_* m_* K_0(C(X)) + i_* K_0(\mathfrak{J}/\mathfrak{K}).$$

Here $m : C(X) \rightarrow \mathfrak{A}$ sends f to the multiplication operator $\begin{pmatrix} f & 0 \\ 0 & 0 \end{pmatrix}$ and i is the inclusion $\mathfrak{I}/\mathfrak{K} \rightarrow \mathfrak{A}/\mathfrak{K}$; m_* and i_* are then the corresponding induced maps in K_0 . We will in general suppress i and i_* to simplify notation.

We will show that $\widetilde{\text{res}}_*$ vanishes on both terms in the right hand side of (3.1). The following lemma treats the first of these terms:

Lemma 1. *$\widetilde{\text{res}}_*$ vanishes on $q_* m_* K_0(C(X))$.*

Proof. Recall that a multiplication operator is in particular a Green operator whose noncommutative residue is zero.

Let $f \in \mathcal{IP}_m(C^\infty(X))$; $m(f)$ acts by multiplication with a smooth (matrix) function and therefore lies in $\mathcal{IP}_m(\mathcal{A})$. Then $q_* m_* [f]_0 = q_* [m(f)]_0 = [m(f) + \mathfrak{K}]_0$, and according to (2.7)

$$(3.2) \quad \widetilde{\text{res}}_*(q_* m_* [f]_0) = \text{res}_*[m(f)]_0 = \text{res}_X(m(f)) = 0.$$

Since $C^\infty(X)$ is local in $C(X)$ [1, 3.1.1-2], any element of $K_0(C(X))$ can be written as $[f]_0 - [g]_0$ for some $f, g \in \mathcal{IP}_m(C^\infty(X))$, cf. (2.3). The lemma follows from this. \square

We now turn to the second term of (3.1); our strategy is to show that the elements of $K_0(\mathfrak{I}/\mathfrak{K})$ correspond to ψ dos with symbols supported in the interior of X . This allows us to construct certain projections for which the noncommutative residue is given as the residue of a projection on the closed manifold \tilde{X} .

The principal symbol induces an isomorphism $\mathfrak{I}/\mathfrak{K} \cong C_0(S^*X^\circ)$ [10, Theorem 1]. We will denote the induced isomorphism in K_0 by σ_* , i.e.,

$$(3.3) \quad \sigma_* : K_0(\mathfrak{I}/\mathfrak{K}) \xrightarrow{\cong} K_0(C_0(S^*X^\circ)).$$

Like in Lemma 1 we wish to consider smooth functions instead of merely continuous functions; the following shows that instead of $C_0(S^*X^\circ)$, it suffices to look at smooth functions (symbols) compactly supported in the interior:

The algebra $C_c^\infty(S^*X^\circ)$, equipped with the sup-norm, is a local C^* -algebra [1, 3.1.1-2] with completion $C_0(S^*X^\circ)$. It follows from (2.2) that the injection $C_c^\infty(S^*X^\circ) \rightarrow C_0(S^*X^\circ)$ induces an isomorphism

$$(3.4) \quad K_0(C_c^\infty(S^*X^\circ)) \cong K_0(C_0(S^*X^\circ)).$$

We now show that each compactly supported symbol in $K_0(C_c^\infty(S^*X^\circ))$ gives rise to a ψ do projection Π_+ on X which is in fact the truncation of a

ψ do projection on \tilde{X} . This will allow us to calculate the residue of Π_+ from the residue of a projection on the closed manifold \tilde{X} .

Lemma 2. *Let $p(x, \xi) \in \mathcal{IP}_m(C_c^\infty(S^*X^\circ)^\sim)$. There is a zero-order ψ do projection Π acting on $C^\infty(X, \mathbb{C}^m)$, such that its symbol is constant on $\tilde{X} \setminus X$, its truncation Π_+ is an idempotent in $\mathcal{M}_m(\mathcal{I}^\sim)$, and*

$$(3.5) \quad \sigma_* q_*([\Pi_+]_0) = [p]_0.$$

Proof. By definition of the unitization of $C_c^\infty(S^*X^\circ)$, we can write p as a sum

$$(3.6) \quad p(x, \xi) = \alpha(x, \xi) + \beta,$$

with $\alpha \in \mathcal{M}_m(C_c^\infty(S^*X^\circ))$ and $\beta \in \mathcal{M}_m(\mathbb{C})$. Note that β itself is idempotent, since $p = \beta$ outside the support of α .

We extend α by zero to obtain a smooth function on the closed manifold $S^*\tilde{X}$ denoted $\tilde{\alpha}(x, \xi)$. We get a ψ do symbol (also denoted $\tilde{\alpha}(x, \xi)$) of order zero on \tilde{X} by requiring $\tilde{\alpha}$ to be homogeneous of degree zero in ξ . Let $\tilde{p}(x, \xi) = \tilde{\alpha}(x, \xi) + \beta$.

We now have an idempotent ψ do-symbol \tilde{p} on \tilde{X} ; we then construct a ψ do projection on \tilde{X} that has \tilde{p} as its principal symbol.

In [7, Chapter 3], Grubb constructed an operator that, for a suitable choice of atlas on the manifold, carries over to the Euclidean Laplacian in each chart, modulo smoothing operators. Hence, choose that particular atlas on \tilde{X} and let D denote this particular operator, i.e., with scalar symbol $d(x, \xi) = |\xi|^2$. Define the auxiliary second order ψ do $C = \text{OP}(c(x, \xi))$, with symbol $c(x, \xi)$ given in the local coordinates of the specified charts as

$$(3.7) \quad c(x, \xi) = (2\tilde{p}(x, \xi) - I)d(x, \xi).$$

Since \tilde{p} is idempotent, the eigenvalues of $2\tilde{p} - I$ are ± 1 , cf. (A.2), so C is an elliptic second order operator and $c(x, \xi) - \lambda$ is parameter-elliptic for λ on each ray in $\mathbb{C} \setminus \mathbb{R}$.

Then we can define the sectorial projection, cf. [12], [5], $\Pi = \Pi_{\theta, \varphi}(C)$ with angles $\theta = -\frac{\pi}{2}$, $\varphi = \frac{\pi}{2}$,

$$(3.8) \quad \Pi = \frac{i}{2\pi} \int_{\Gamma_{\theta, \varphi}} \lambda^{-1} C(C - \lambda)^{-1} d\lambda.$$

Π is a ψ do projection [12] on \tilde{X} with symbol π given in local coordinates by

$$(3.9) \quad \pi(x, \xi) = \frac{i}{2\pi} \int_{C(x, \xi)} q(x, \xi, \lambda) d\lambda,$$

where $q(x, \xi, \lambda)$ is the symbol with parameter for a parametrix of $c(x, \xi) - \lambda$, and $\mathcal{C}(x, \xi)$ is a closed curve encircling the eigenvalues of $c_2(x, \xi)$ — the principal symbol of C — in the $\{\operatorname{Re} z > 0\}$ half-plane.

The eigenvalues of $c_2(x, \xi) = (2\tilde{p}(x, \xi) - I)|\xi|^2$ are $\pm|\xi|^2$, so we can choose $\mathcal{C}(x, \xi)$ as the boundary of a small ball $B(|\xi|^2, r)$ around $+\lambda^2$.

Hence, the principal symbol of $\pi(x, \xi)$ is

$$\begin{aligned} \pi_0(x, \xi) &= \frac{i}{2\pi} \int_{\mathcal{C}(x, \xi)} q_{-2}(x, \xi, \lambda) d\lambda \\ (3.10) \quad &= \frac{i}{2\pi} \int_{\partial B(|\xi|^2, r)} [(2\tilde{p}(x, \xi) - I)|\xi|^2 - \lambda]^{-1} d\lambda = \tilde{p}(x, \xi), \end{aligned}$$

according to Lemma 4. So Π is a ψ do projection with principal symbol $\tilde{p}(x, \xi)$, as desired.

Observe that for x outside the support of $\tilde{\alpha}$, we have $c(x, \xi) = (2\beta - I)|\xi|^2$ and $q(x, \xi, \lambda) = q_{-2}(x, \xi, \lambda) = ((2\beta - I)|\xi|^2 - \lambda)^{-1}$ so $\pi(x, \xi) = \pi_0(x, \xi) = \beta$ there. (We cannot be sure that the full symbol of π equals \tilde{p} inside the support, since coordinate-dependence will in general influence the lower order terms of the parametrix.) In particular, $\pi(x, \xi)$ is constant equal to β for $x \in \tilde{X} \setminus X$.

Now consider the truncation Π_+ . We have

$$(3.11) \quad (\Pi_+)^2 = (\Pi^2)_+ - L(\Pi, \Pi) = \Pi_+ - L(\Pi, \Pi),$$

where the singular Green operator $L(P, Q)$ is defined as $(PQ)_+ - P_+Q_+$ for ψ dos P and Q . Since $\pi(x, \xi)$ equals the constant matrix β in a neighborhood of the boundary ∂X it follows, cf. [6, Theorem 2.7.5], that $L(\Pi, \Pi) = 0$, so $(\Pi_+)^2 = \Pi_+$.

Since the symbol of $\Pi - \beta$ is compactly supported within X° , we can write $\Pi_+ = \varphi P\psi + \beta$ for some φ, ψ, P , as in (2.5); hence Π_+ is in $\mathcal{M}_m(\mathcal{I}^\sim)$. Technically, Π_+ lies in the algebra where the boundary bundle F is the zero-bundle, but inserting zeros into Π_+ 's matrix form will clearly allow us to augment it to the present case with $F = \partial X \times \mathbb{C}$.

Finally we take a look at (3.5): Since Π_+ is an idempotent in $\mathcal{M}_m(\mathcal{I}^\sim)$ it defines a K_0 -class $[\Pi_+]_0$ in $K_0(\mathcal{I}^\sim)$. Then $q_*[\Pi_+]_0$ defines a class in $K_0(\mathcal{I}/\mathcal{K}^\sim)$, a class defined solely by its principal symbol. Since the principal symbol is exactly the idempotent $p(x, \xi)$ we obtain (3.5) by definition. \square

Theorem 3. *The noncommutative residue of any projection in (the norm closure of) the Boutet de Monvel calculus is zero.*

Proof. As mentioned, it suffices to show that res_* vanishes on $K_0(\mathcal{A}) \cong K_0(\mathfrak{A})$. In turn, according to equation (3.1) and Lemma 1, we only need to show that $\widetilde{\text{res}}_*$ vanishes on $K_0(\mathfrak{J}/\mathfrak{K})$.

So let $\omega \in K_0(\mathfrak{J}/\mathfrak{K})$. Employing (2.4), (3.3), and (3.4) we can find p, p' in $\mathcal{IP}_m(C_c^\infty(S^*X^\circ)^\sim)$ such that

$$(3.12) \quad \sigma_*\omega = [p]_0 - [p']_0.$$

Now, for p, p' we use Lemma 2 to find corresponding ψ do's Π, Π' with the specific properties mentioned there. By (3.5) and (3.12) we see that

$$(3.13) \quad q_*[\Pi_+]_0 - q_*[\Pi'_+]_0 = \sigma_*^{-1}([p]_0 - [p']_0) = \omega.$$

Using equation (2.7) we now see that

$$(3.14) \quad \widetilde{\text{res}}_*\omega = \text{res}_X(\Pi_+) - \text{res}_X(\Pi'_+).$$

Here

$$(3.15) \quad \text{res}_X(\Pi_+) = \int_X \int_{S_x^*X} \text{tr } \pi_{-n}(x, \xi) dS(\xi) dx.$$

By construction, $\pi(x, \xi)$ is constant equal to β outside X ; in particular $\pi_{-n}(x, \xi)$ is zero for $x \in \widetilde{X} \setminus X$ and therefore

$$(3.16) \quad \int_X \int_{S_x^*X} \text{tr } \pi_{-n}(x, \xi) dS(\xi) dx = \int_{\widetilde{X}} \int_{S_x^*\widetilde{X}} \text{tr } \pi_{-n}(x, \xi) dS(\xi) dx.$$

In other words

$$(3.17) \quad \text{res}_X(\Pi_+) = \text{res}_{\widetilde{X}}(\Pi),$$

where the latter is the noncommutative residue of a ψ do projection on a closed manifold. It is well-known [14], [15] that the latter always vanishes. Likewise we obtain $\text{res}_X(\Pi'_+) = 0$ and finally

$$(3.18) \quad \widetilde{\text{res}}_*\omega = 0$$

as desired. \square

In [5], it was an open question whether the residue is zero on a sectorial projection for a boundary value problem. This theorem answers that question in the positive for the cases where the sectorial projection lies in the C^* -closure of \mathcal{A} .

It is not, at this time, clear for which boundary value problems this is true. We showed in [5] that there certainly are boundary value problems where the sectorial projection is not in \mathcal{A} ; whether or not they lie in \mathfrak{A} is something we intend to return to in a future work.

A. APPENDIX

Lemma 4. *Let $M \in \mathcal{IP}_m(\mathbb{C})$. Let $d > 0$ and let $\partial B(d, r)$ denote the closed curve in the complex plane along the boundary of the ball with center d and radius $0 < r < d$. Then*

$$(A.1) \quad \frac{i}{2\pi} \int_{\partial B(d, r)} [(2M - I)d - \lambda]^{-1} d\lambda = M.$$

Proof. A direct computation shows that, for $\lambda \neq \pm d$,

$$(A.2) \quad [(2M - I)d - \lambda]^{-1} = \frac{M}{d - \lambda} - \frac{I - M}{d + \lambda}.$$

The result in (A.1) then follows from the residue theorem. \square

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REFERENCES

- [1] Bruce Blackadar, *K-theory for operator algebras*, second ed., Mathematical Sciences Research Institute Publications, vol. 5, Cambridge University Press, Cambridge, 1998.
- [2] Louis Boutet de Monvel, *Boundary problems for pseudo-differential operators*, Acta Math. **126** (1971), no. 1-2, 11–51.
- [3] Tamar Burak, *On spectral projections of elliptic operators*, Ann. Scuola Norm. Sup. Pisa (3) **24** (1970), 209–230.
- [4] Boris V. Fedosov, François Golse, Eric Leichtnam, and Elmar Schrohe, *The non-commutative residue for manifolds with boundary*, J. Funct. Anal. **142** (1996), no. 1, 1–31.
- [5] Anders Gaarde and Gerd Grubb, *Logarithms and sectorial projections for elliptic boundary problems*, 2007, arXiv:math/0703878. To appear in Math. Scand.
- [6] Gerd Grubb, *Functional calculus of pseudodifferential boundary problems*, second ed., Progress in Mathematics, vol. 65, Birkhäuser Boston Inc., Boston, MA, 1996.
- [7] ———, *The local and global parts of the basic zeta coefficient for operators on manifolds with boundary*, 2007, arXiv:math/0611854. To appear in Math. Ann.
- [8] Gerd Grubb and Elmar Schrohe, *Trace expansions and the noncommutative residue for manifolds with boundary*, J. Reine Angew. Math. **536** (2001), 167–207.
- [9] Victor Guillemin, *A new proof of Weyl's formula on the asymptotic distribution of eigenvalues*, Adv. in Math. **55** (1985), no. 2, 131–160.
- [10] Severino T. Melo, Ryszard Nest, and Elmar Schrohe, *C^* -structure and K -theory of Boutet de Monvel's algebra*, J. Reine Angew. Math. **561** (2003), 145–175.
- [11] Severino T. Melo, Thomas Schick, and Elmar Schrohe, *A K -theoretic proof of Boutet de Monvel's index theorem for boundary value problems*, J. Reine Angew. Math. **599** (2006), 217.

- [12] Raphaël Ponge, *Spectral asymmetry, zeta functions, and the noncommutative residue*, Internat. J. Math. **17** (2006), no. 9, 1065–1090.
- [13] Elmar Schrohe, *A short introduction to Boutet de Monvel's calculus*, Approaches to singular analysis (Berlin, 1999), Oper. Theory Adv. Appl., vol. 125, Birkhäuser, Basel, 2001, pp. 85–116.
- [14] Mariusz Wodzicki, *Spectral asymmetry and zeta functions*, Invent. Math. **66** (1982), no. 1, 115–135.
- [15] ———, *Local invariants of spectral asymmetry*, Invent. Math. **75** (1984), no. 1, 143–177.

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